[10-01-12A-T11]

Solutions to selected problems from H16C

[H16C 2b]

Compute $\left(1 - i\sqrt{3}\right)^{11}$

$r = \sqrt{1+3} = 2$

$\tan\theta = -\sqrt{3} \Longrightarrow \theta = -\frac{\pi}{3}$, because $\left(1 - i\sqrt{3}\right)$ is in quadrant IV

$\left(1 - i\sqrt{3}\right)^{11} = 2^{11}\,(\text{cis } -\frac{\pi}{3})^{11} = 2^{11}\text{ cis }\frac{-11\pi}{3} = 2^{11}\text{ cis }\frac{\pi}{3}$.

Then,

$2^{11}\text{ cis }\frac{\pi}{3} = 2^{11}\,(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 2^{11}\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = 2^{10}\left(1 + i\sqrt{3}\right)$

$\therefore \left(1 - i\sqrt{3}\right)^{11} = 1024 + 1024\,i$

[H16C 2c]

Compute $\left(\sqrt{2} - i\sqrt{2}\right)^{-19}$

$r = \sqrt{2+2} = 2$

$\tan\theta = -1 \Longrightarrow \theta = \frac{-\pi}{4}$, because $\left(\sqrt{2} - i\sqrt{2}\right)$ is in quadrant IV

$\left(\sqrt{2} - i\sqrt{2}\right)^{-19} = 2^{-19}\,(\text{cis }\frac{-\pi}{4})^{-19} = 2^{-19}\text{ cis }\frac{-19\pi}{4} = 2^{-19}\text{ cis }\frac{3\pi}{4}$.

Then,

$2^{-19}\text{ cis }\frac{3\pi}{4} = 2^{-19}\,(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}) = 2^{-19}\left(\frac{-\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right) = 2^{-19}\,2^{-1/2}\,(-1 + i)$

$\therefore \left(\sqrt{2} - i\sqrt{2}\right)^{-19} = 2^{-39/2}\,(-1 + i)$. {this is equal to $(\frac{1}{524288})\left(\frac{-\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)$ the book answer.}

OVER ⟶

[H16C 11a]

If $z^n = \text{cis}\,\theta$, prove $z^n + \frac{1}{z^n} = 2\cos(n\,\theta)$.

Proof.

Let $z^n = \text{cis}\,\theta$. Then $z^n + \frac{1}{z^n} = (\text{cis}\,\theta)^n + (\text{cis}\,\theta)^{-n}$.

Now $(\text{cis}\,\theta)^{-n} = \cos(-n\,\theta) + i\sin(-n\,\theta) = \cos(n\,\theta) - i\sin(n\,\theta)$.

So that $(\text{cis}\,\theta)^n + (\text{cis}\,\theta)^{-n} = \cos(n\,\theta) + i\sin(n\,\theta) + \cos(n\,\theta) - i\sin(n\,\theta) = 2\cos(n\,\theta)$. □